Eur. Phys. J. A **19**, 139–144 (2004)
DOI 10.1140/epja/i2003-10112-5

THE EUROPEAN PHYSICAL JOURNAL A

SU(2) × SU(2) nonlocal quark model with confinement

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Received: 16 June 2003 / Revised version: 28 July 2003 /
Published online: 18 December 2003 –
Communicated by V.V. Anisovich

Abstract. The nonlocal version of the $SU(2) \times SU(2)$ symmetric four-quark interaction of the NJL type is considered. Each of the quark lines contains the form factors. These form factors remove the ultraviolet divergences in quark loops. The additional condition on the quark mass function $m(p)$ ensures the absence of the poles in the quark propagator (quark confinement). The constituent-quark mass $m(0)$ is expressed through the cut-off parameter Λ, $m(0) = \Lambda = 340$ MeV in the chiral limit. These parameters are fixed by the experimental value of the weak pion decay and allow us to describe the mass of the light scalar meson, the strong decay $\rho \to \pi\pi$ and the D/S ratio in the decay $a_1 \to \rho\pi$ in satisfactory agreement with the experimental data.

PACS. 14.40.-n Mesons – 11.10.Lm Nonlinear or nonlocal theories and models – 12.39.Ki Relativistic quark model

1 Introduction

The effective meson Lagrangians obtained on the basis of the local four-quark interaction of the Nambu–Jona-Lasinio (NJL) type satisfactorily describe low-energy meson physics [1–4]. However, these models contain ultraviolet (UV) divergences and do not describe quark confinement. Therefore, additional regularization is necessary in these models. Besides, it is impossible to take into account the dependence of the amplitude of different processes on large external momentum in order to provide quark confinement. This situation restricts the predictive power of these models. Satisfactory results in these models can be obtained only for light mesons and interactions at low energies in the range of 1 GeV. In order to overcome these restrictions, it is necessary to consider nonlocal versions of these models which allow us to remove UV divergences and describe the quark confinement.

A lot of models of this type were proposed in the last few years. Unfortunately, we cannot give here the full list of references concerning this activity. Therefore, we will concentrate only on the direction connected with the nonlocal quark interaction motivated by the instanton theories [5–8]. Recently, a few nonlocal models of this type were proposed [9–12]. In these models the nonlocal kernel is taken in the separable form where each quark line contains form factors following from instanton theories. These form factors naturally remove UV divergences in quark loops. Thus, in [9,10] a nonlocal form factor was chosen in the Gaussian form $f(p) = \exp(-p^2/\Lambda^2)$, where Λ is the cut-off parameter[1]. In [11,12] it was proposed to use an additional condition for the form factor $f(p)$ (quark mass function $m(p)$, respectively) which leads to the absence of the poles in the quark propagator. Namely, it is supposed that the scalar part of the quark propagator is expressed through the entire function

$$\frac{m(p)}{p^2 + m^2(p)} = \frac{1}{\mu} \exp\left(-p^2/\Lambda^2\right), \tag{1}$$

where μ is an additional arbitrary parameter. The similar condition providing confinement was used in [13–15].

In this work an analogous condition will be used. However, we will take into account that each quark line contains the square of the form factor which is expressed through the quark mass

$$\frac{m(p)f^2(p)}{p^2 + m^2(p)} \to \frac{m^2(p)}{p^2 + m^2(p)} = \exp\left(-p^2/\Lambda^2\right). \tag{2}$$

As a result, we obtain a simpler solution for the mass function than in [11,12]. In our model $m(0)$ and the cut-off parameter Λ have a simple connection in the chiral limit $m(0) = \Lambda$; the function $m(p)$ contains only one arbitrary

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[1] Here and further all expressions are given in Euclidean domain.

parameter. We fix this parameter by weak pion decay. Then, for $F_\pi = 93$ MeV we have $m(0) = \Lambda = 340$ MeV. This leads to reasonable predictions for the scalar-meson mass, the width of the decay $\rho \to \pi\pi$ and the D/S ratio in the decay $a_1 \to \rho\pi$, where D, S are the partial waves of this decay.

The paper is organized as follows. In sect. 2, we consider a nonlocal four-quark interaction and after bosonization derive the gap equation for the dynamical quark mass. The additional condition for this mass allows us to provide the quark confinement. In sect. 3, the masses and couplings of the scalar and pseudoscalar mesons are obtained and the main parameters of the model are fixed. The decay width $\sigma \to \pi\pi$ is calculated. In sect. 4, the vector and axial-vector sectors of the model are considered. The a_1-meson mass, decay widths $\rho \to \pi\pi$, $a_1 \to \rho\pi$ are calculated. The D/S ratio in the decay $a_1 \to \rho\pi$ is estimated. The π-a_1 mixing is studied. The discussion of the obtained results and comparison with other models is given in the last section.

2 SU(2) × SU(2) nonlocal quark interaction

The $SU(2) \times SU(2)$ symmetric action with the nonlocal four-quark interaction has the form

$$\begin{aligned}\mathcal{S}(\bar q, q) = \int \mathrm{d}^4x \Big\{ &\bar q(x)(i\hat\partial_x - m_\mathrm{c})q(x) \\ &+\frac{G_1}{2}\left(J^a_\pi(x)J^a_\pi(x) + J_\sigma(x)J_\sigma(x)\right) \\ &-\frac{G_2}{2}\left(J^{\mu\,a}_\rho(x)J^{\mu\,a}_\rho(x) + J^{\mu\,a}_{a_1}(x)J^{\mu\,a}_{a_1}(x)\right)\Big\},\end{aligned} \tag{3}$$

where $\bar q(x) = (\bar u(x), \bar d(x))$ are the u- and d-quark fields, m_c is the diagonal matrix of the current quark masses, G_1 is the coupling constant of the scalar- and pseudoscalar-quark currents, G_2 is the coupling constant of the vector and axial-vector quark currents. The nonlocal quark currents $J_I(x)$ are expressed as

$$J_I(x) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 f(x_1)f(x_2)\bar q(x-x_1)\,\Gamma_I\, q(x+x_2), \tag{4}$$

where $f(x)$ are the nonlocal functions. In (4) the matrices Γ_I are defined as

$$\Gamma_\sigma = \mathbf{1}, \quad \Gamma^a_\pi = i\gamma^5\tau^a, \quad \Gamma^{\mu\,a}_\rho = \gamma^\mu\tau^a, \quad \Gamma^{\mu\,a}_{a_1} = \gamma^5\gamma^\mu\tau^a,$$

where τ^a are the Pauli matrices and γ^μ, γ^5 are the Dirac matrices.

In this article, we mainly consider the strong interactions. The electroweak fields may be introduced by gauging the quark field by the Schwinger phase factors (see [8–10]).

After bosonization the action becomes

$$\begin{aligned}\mathcal{S}(q,\bar q,\sigma,\pi,\rho,a) = \int \mathrm{d}^4x \Big\{ &-\frac{1}{2G_1}\left(\pi^a(x)^2 + \tilde\sigma(x)^2\right) \\ &+\frac{1}{2G_2}\left((\rho^{\mu\,a}(x))^2 + (a_1^{\mu\,a}(x))^2\right) + \bar q(x)(i\hat\partial_x - m_\mathrm{c})q(x) \\ &+\int \mathrm{d}^4x_1\mathrm{d}^4x_2\, f(x-x_1)f(x_2-x)\,\bar q(x_1)\left(\pi^a(x)i\gamma^5\tau^a\right. \\ &\left.+\tilde\sigma(x) + \rho^{\mu\,a}(x)\gamma^\mu\tau^a + a_1^{\mu\,a}(x)\gamma^5\gamma^\mu\tau^a\right) q(x_2)\Big\},\end{aligned} \tag{5}$$

where $\tilde\sigma, \pi, \rho, a$ are the σ-, π-, ρ-, a_1-meson fields, respectively. The field $\tilde\sigma$ has a nonzero vacuum expectation value $\langle\tilde\sigma\rangle_0 = \sigma_0 \neq 0$. In order to obtain a physical scalar field with zero vacuum expectation value, it is necessary to shift the scalar field as $\tilde\sigma = \sigma + \sigma_0$. This leads to the appearance of the quark mass function $m(p)$ instead of the current quark mass m_c

$$m(p) = m_\mathrm{c} + m_\mathrm{dyn}(p), \tag{6}$$

where $m_\mathrm{dyn}(p) = -\sigma_0 f^2(p)$ is the dynamical quark mass. From the action, eq. (5), by using

$$\left\langle \frac{\delta S}{\delta\sigma} \right\rangle_0 = 0,$$

one can obtain the gap equation for the dynamical quark mass

$$m_\mathrm{dyn}(p) = G_1 \frac{8N_c}{(2\pi)^4} f^2(p) \int \mathrm{d}^4_E k f^2(k)\frac{m(k)}{k^2 + m^2(k)}. \tag{7}$$

The right-hand side of this equation is the tadpole of the quark propagator taken in the Euclidean domain. Equations (6), (7) have the following solution:

$$m(p) = m_\mathrm{c} + (m_q - m_\mathrm{c})f^2(p), \tag{8}$$

where $m_q = m(0)$.

In order to provide quark confinement we propose the following anzatz for the quark mass function $m(p)$. We suppose that mass satisfies the following condition in the chiral limit:

$$\frac{m^2(p)}{m^2(p) + p^2} = \exp\left(-p^2/\Lambda^2\right). \tag{9}$$

The form of the left-hand side of this equation coincides with the integrand in the gap equation (7). From eq. (9) we obtain the following solution[2]:

$$m(p) = \left(\frac{p^2}{\exp\left(p^2/\Lambda^2\right) - 1}\right)^{1/2}; \tag{10}$$

here we have only one free parameter Λ; $m(p)$ does not have any singularities in the whole real axis and exponentially drops as $p^2 \to \infty$ in the Euclidean domain. From

[2] Here only the positive solution will be used.

eq. (8) it follows that the form factors have a similar behavior that provides the absence of UV divergences in our model. At $p^2 = 0$ the mass function is equal to the cutoff parameter Λ, $m(0) = \Lambda$. The pole part of the quark propagator also does not contain singularities that provide quark confinement[3]

$$\frac{1}{m^2(p) + p^2} = \frac{1 - \exp\left(-p^2/\Lambda^2\right)}{p^2}. \qquad (11)$$

When taking into account the current quark mass, eq. (9) can be modified as follows:

$$\frac{m^2(p) - m_c^2}{m^2(p) + p^2} = \exp\left(-\left(p^2 + m_c^2\right)/\Lambda^2\right). \qquad (12)$$

Here, m_c^2 is introduced in the form that conserves the analytical properties of the mass function $m(p)$. Then the mass function takes the form

$$m(p) = \left(\frac{m_c^2 + p^2 \exp\left(-\left(p^2 + m_c^2\right)/\Lambda^2\right)}{1 - \exp\left(-(p^2 + m_c^2)/\Lambda^2\right)}\right)^{1/2}. \qquad (13)$$

3 Pseudoscalar and scalar mesons

Let us consider the scalar and pseudoscalar mesons. The meson propagators are given by

$$D_{\sigma,\pi}(p^2) = \frac{1}{-G_1^{-1} + \Pi_{\sigma,\pi}(p^2)} = \frac{g^2_{\sigma,\pi}(p^2)}{p^2 - M^2_{\sigma,\pi}}, \qquad (14)$$

where $M_{\sigma,\pi}$ are the meson masses, $g_{\sigma,\pi}(p^2)$ are the functions describing the renormalization of the meson fields and $\Pi_{\sigma,\pi}(p^2)$ are the polarization operators defined by

$$\Pi_{\sigma,\pi}(p^2) = i\frac{2N_c}{(2\pi)^4}\int \mathrm{d}^4k f^2(k_-^2) f^2(k_+^2) \times \mathrm{Sp}\left[\mathrm{S}(k_-)\Gamma_{\sigma,\pi}\mathrm{S}(k_+)\Gamma_{\sigma,\pi}\right], \qquad (15)$$

where $k_\pm = k \pm p/2$.

For the calculation of these integrals it is necessary to rewrite these expressions in the Euclidean space where the form factors (and quark masses) are the exponentially decreasing functions. Then eq. (15) takes the form

$$\Pi_{\sigma,\pi}(p^2) = \frac{2N_c}{(2\pi)^4 m_q^2}\int \mathrm{d}^4_E k \times \frac{P_{\sigma,\pi}(k^2, p^2, p\cdot k)}{(k_+^2 + m(k_+^2)^2)(k_-^2 + m(k_-^2)^2)}. \qquad (16)$$

The functions $P_{\sigma,\pi}(k^2, p^2, p\cdot k)$ are the Dirac trace multiplied by $m(k_+)$, $m(k_-)$. In eq. (16) all momenta are Euclidean. In the description of the meson properties it is necessary to make the analytical continuation of this expression over external momenta p to the Minkowski space.

[3] Note that similar functions were used in [14–16] in order to describe the quark confinement.

Let us emphasize that at our anzatz for a quark mass function only the functions $P_{\sigma,\pi}(k^2, p^2, p\cdot k)$ contains nonanalytical root cuts, whereas there are no problems with the analytical continuation of the denominator.

The meson masses $M_{\sigma,\pi}$ are found from the position of the pole in the meson propagator

$$\Pi_{\sigma,\pi}(M^2_{\sigma,\pi}) = G_1^{-1}, \qquad (17)$$

and the constants $g_{\sigma,\pi}(M^2_{\sigma,\pi})$ are given by

$$g^{-2}_{\sigma,\pi}(M^2_{\sigma,\pi}) = \frac{\mathrm{d}\Pi_{\sigma,\pi}(p^2)}{\mathrm{d}p^2}\Big|_{p^2 = M^2_{\sigma,\pi}}. \qquad (18)$$

Firstly, let us consider this model in the chiral limit. The pion constant $g_\pi(0)$ does not depend on the parameter Λ and takes the form

$$g_\pi^{-2}(0) = \frac{N_c}{4\pi^2}\left(\frac{3}{8} + \frac{\zeta(3)}{2}\right), \quad g_\pi(0) \approx 3.7; \qquad (19)$$

here ζ is the Riemann zeta-function.

The gap equation has the simple form

$$G_1 \Lambda^2 = \frac{2\pi^2}{N_c}. \qquad (20)$$

The quark condensate is

$$\langle \bar{q}q \rangle_0 = -\frac{N_c}{4\pi^2}\int_0^\infty \mathrm{d}u\, u \frac{m(u)}{u + m^2(u)}. \qquad (21)$$

The Goldberger-Treiman relation is fulfilled in the model of this kind [9,10,8,12]

$$F_\pi = \frac{m_q}{g_\pi}. \qquad (22)$$

From eqs. (19), (22) the value $\Lambda = m_q = 340$ MeV is obtained for $F_\pi = 93$ MeV. Then, from eqs. (20), (21) we obtain

$$G_1 = 56.6\,\mathrm{GeV}, \quad \langle \bar{q}q \rangle_0 = -(188\,\mathrm{MeV})^3. \qquad (23)$$

In the description of the pion mass it is necessary to introduce the nonzero current quark mass m_c. In our model $M_\pi^2 \ll \Lambda^2$. Therefore, we can consider only the lowest order of the expansion in small p^2. Then, one gets from eq. (14)

$$M_\pi^2 = g_\pi^2(0)\left(G_1^{-1} - \frac{N_c}{2\pi^2}\int_0^\infty \mathrm{d}u\, u \frac{f(u)^4}{u + m^2(u)}\right). \qquad (24)$$

By using the expression for G_1 from the gap equation (7), the Gell-Mann–Oakes–Renner relation can be reproduced:

$$M_\pi^2 = -2\frac{m_c \langle \bar{q}q \rangle_0}{F_\pi^2} + O(m_c^2). \qquad (25)$$

From eq. (25) with $M_\pi = 140$ MeV we can estimate the value of the current quark mass $m_c \approx 13$ MeV. Other model parameters in this case change very little

$$\Lambda = 343\,\mathrm{MeV}, \quad g_\pi(M_\pi) = 3.57,$$
$$G_1 = 56.5\,\mathrm{GeV}, \quad \langle \bar{q}q \rangle_0 = -(189\,\mathrm{MeV})^3. \tag{26}$$

Therefore, in calculations of the amplitudes of various processes we can use the values of parameters taken in the chiral limit.

With the help of the parameters (23) we get for the sigma-meson $M_\sigma = 420$ MeV and $g_\sigma(M_\sigma) = 3.85$. The amplitude of the decay $\sigma \to \pi\pi$ is equal to $A_{(\sigma\to\pi^+\pi^-)} = 1.67$ GeV. Then, the total decay width is $\Gamma_{(\sigma\to\pi\pi)} = 150$ MeV. Comparing these results with experimental data one finds that M_σ is in satisfactory agreement with experiment $M_\sigma^{\mathrm{exp}} = 400$–$1200$; however, the decay width is very small $\Gamma_\sigma^{\mathrm{exp}} = 600$–$1000$.

4 Vector and axial-vector mesons

The propagators of the vector and axial-vector mesons have the transversal and longitudinal parts

$$D^{\mu\nu}_{\rho,a_1} = T^{\mu\nu} D^{\mathrm{T}}_{\rho,a_1} + L^{\mu\nu} D^{\mathrm{L}}_{\rho,a_1}, \tag{27}$$

where $T^{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu/p^2$, $L^{\mu\nu} = p^\mu p^\nu/p^2$ and

$$D^{\mathrm{T}}_{\rho,a_1} = \frac{1}{G_2^{-1} + \Pi^{\mathrm{T}}_{\rho,a_1}(p^2)} = \frac{g^2_{\rho,a_1}(p^2)}{M^2_{\rho,a_1} - p^2},$$
$$D^{\mathrm{L}}_{\rho,a_1} = \frac{1}{G_2^{-1} + \Pi^{\mathrm{L}}_{\rho,a_1}(p^2)}. \tag{28}$$

Here, $\Pi^{\mathrm{T}}_{\rho,a_1}$ and $\Pi^{\mathrm{L}}_{\rho,a_1}$ are the transversal and longitudinal parts of the polarization operator $\Pi^{\mu\nu}_{\rho,a_1}(p^2)$:

$$\Pi^{\mu\nu}_{\rho,a_1}(p^2) = i\frac{2N_c}{(2\pi)^4}\int \mathrm{d}^4k f^2(k_-)f^2(k_+)$$
$$\times \mathrm{Sp}\left[\mathrm{S}(k_-)\Gamma_{\rho,a_1}\mathrm{S}(k_+)\Gamma_{\rho,a_1}\right].$$

The constant G_2 is fixed by the ρ-meson mass

$$G_2^{-1} = -\Pi^{\mathrm{T}}_\rho(M_\rho)$$

and $G_2 = 6.5$ GeV^{-2}. Then the a_1-meson mass is equal to 970 MeV.

The constants $g_{\rho,a_1}(M^2_{\rho,a_1})$ are equal to

$$g^{-2}_{\rho,a_1}(M^2_{\rho,a_1}) = -\frac{\mathrm{d}\Pi^{\mathrm{T}}_{\rho,a_1}(p^2)}{\mathrm{d}p^2}\Big|_{p^2=M^2_{\rho,a_1}}. \tag{29}$$

From eq. (29) we obtain $g_\rho(M_\rho) = 1.23$, $g_{a_1}(M_{a_1}) = 1.33$. At $p^2 = 0$ we have $g_\rho(0) \approx 2$, $g_{a_1}(0) \approx 2.5$. (see also fig. 1).

The decay $\rho \to \pi\pi$ is described by the triangle quark diagram. The amplitude for the process is

$$A^\mu_{(\rho\to\pi\pi)} = a_{(\rho\to\pi\pi)}(q_1 - q_2)^\mu, \tag{30}$$

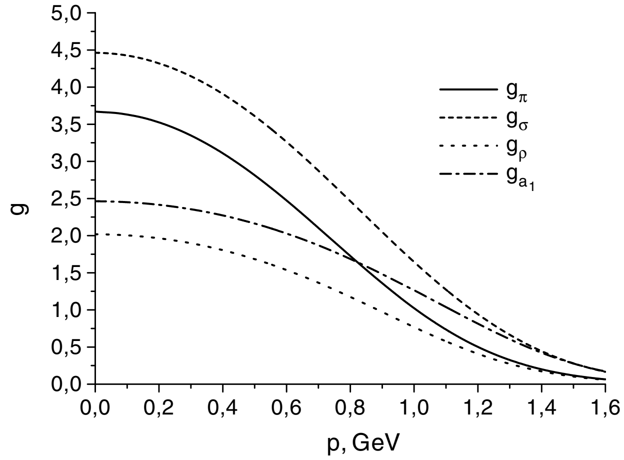

Fig. 1. Momentum dependence of the mesons renormalization functions.

where q_i are momenta of the pions. We obtain $a_{(\rho\to\pi\pi)} = 5.72$ and the decay width $\Gamma_{(\rho\to\pi\pi)} = 135\,\mathrm{MeV}$ which is in qualitative agreement with the experimental value 149.2 ± 0.7 MeV [17].

The decay $a_1 \to \rho\pi$ is described in a similar manner. The amplitude for the process $a_1 \to \rho\pi$ is

$$A^{\mu\nu}_{(a_1\to\rho\pi)} = a_{(a_1\to\rho\pi)}g^{\mu\nu} + b_{(a_1\to\rho\pi)}p^\nu q^\mu, \tag{31}$$

where p, q are momenta of a_1-, ρ-mesons, respectively. We obtain $a_{(a_1\to\rho\pi)} = 2.68$ GeV, $b_{(a_1\to\rho\pi)} = 16.71$ GeV^{-1}. The amplitude of the decay $a_1 \to \rho\pi$ contains D and S waves. The ratio of these waves has the form (see [10,15]):

$$D/S = -\sqrt{2}\frac{(E_\rho - M_\rho)a_{(a_1\to\rho\pi)} + b_{(a_1\to\rho\pi)}M_{a_1}|\vec{q}\,|^2}{(E_\rho + 2M_\rho)a_{(a_1\to\rho\pi)} + b_{(a_1\to\rho\pi)}M_{a_1}|\vec{q}\,|^2}$$
$$-0.06\,, \tag{32}$$

$$|\vec{q}\,|^2 = \lambda(M^2_{a_1}, M^2_\rho, M^2_\pi)/(2M_{a_1})^2, E^2_\rho = M^2_\rho + |\vec{q}\,|^2\,,$$
$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$

This ratio is in satisfactory agreement with the experimental data $D/S^{\mathrm{exp}} = -0.108\pm 0.016$. The decay width equals $\Gamma_{(a_1\to\rho\pi)} = 90$ MeV. This value is noticeably smaller than experiment 250–600 MeV [17].

The longitudinal component of the a_1-meson field is mixed with the pion. The amplitude describing this mixing has the form

$$A^\mu_{(\pi\to a_1)} = iC_{(\pi\to a_1)}(p^2)p^\mu,$$

where p is the momentum of the pion, and $C_{(\pi\to a_1)}(0)$ in the chiral limit is equal to 190 MeV. The additional pion kinetic term from the π-a_1 mixing is $\Delta L_{\mathrm{kin}} = \Delta \cdot p^2\pi^a(p)^2/2$. In the chiral limit, Δ is as follows:

$$\Delta = \frac{C^2_{(\pi\to a_1)}(0)}{g^2_{a_1}(0)(G_2^{-1} + \Pi^{\mathrm{L}}_{a_1}(0))} \approx C^2_{(\pi\to a_1)}(0)G_2/g^2_{a_1}(0) \approx 0.04. \tag{33}$$

Δ is small; therefore, the effect of the π-a_1 mixing can be neglected.

Table 1. The comparison of the physical results which are obtained in local and nonlocal quark models. In the local NJL model the decay width $\rho \to \pi\pi$ is used for the fitting of the model parameters. Two sets of values corresponding to a different choice of model parameters are given in columns 4, 5 (see table 4 in [10] and table I in [15]).

Quantity	Our model	[2]	[10]		[15]		[17]
M_σ (MeV)	420	570	443.2	465.8			400–1200
$\Gamma_{\sigma\to\pi\pi}$ (MeV)	150	190	108	135.1			600–1000
$\Gamma_{\rho\to\pi\pi}$ (MeV)	135	150	126	114	356	259	149.2 ± 0.7
M_{a_1} (MeV)	970	1030	946.8	1061.5	1340		1230 ± 40
$\Gamma_{a_1\to\rho\pi}$ (MeV)	90	290	44	376.2	4020	385	250–600
D/S	-0.06		-0.048	-0.087	-0.092	-0.075	-0.108 ± 0.016

5 Discussion and conclusion

In this work we have considered the possibility of constructing the $SU(2) \times SU(2)$ symmetric nonlocal chiral quark model providing the absence of UV divergences and quark confinement. These features of the model are specified by the nonlocal kernel which appears in the four-quark interaction. Such a structure of the four-quark interaction can be motivated by the instanton interactions [6–8].

The pseudoscalar, scalar, vector and axial-vector mesons have been considered in the framework of this model. The masses and strong coupling constants of the mesons were described. It was shown that the functions describing the renormalization of the meson fields noticeably decreased at large p^2 in the physical domain (see fig. 1).

Among the satisfactory predictions of the model are the mass of the σ-meson, the decay width $\rho \to \pi\pi$ and the D/S ratio in the decay $a_1 \to \rho\pi$.

However, in the description of the a_1-meson mass and decay widths $\sigma \to \pi\pi$, $a_1 \to \rho\pi$ our results are noticeably smaller than the experimental data. Note that the width of the decay $a_1 \to \rho\pi$ strongly depends on the mass of the a_1-meson. Indeed, for $M_{a_1} = 1.26$ GeV we have $\Gamma_{(a_1\to\rho\pi)} \approx 200$ MeV that is in qualitative agreement with experiment.

It is useful to compare the obtained results with the analogous results obtained in the local NJL model [2] and other nonlocal models with quark interactions of separable type [10,15].

Remind that in the local NJL model the cut-off parameter $\Lambda^{(\mathrm{NJL})} = 1.2$ GeV and the constituent-quark mass $m = 280$ MeV are used. These parameters are fixed by the decays $\pi \to \mu\nu$ ($f_\pi = 93$ MeV) and $\rho \to \pi\pi$ ($g_\rho = 6.14$). These parameters lead to the quark condensate $\langle\bar{q}q\rangle_0 = -(293\,\mathrm{MeV})^3$ and the current quark mass $m_{\mathrm{c}} = 3$ MeV. In the present model $m_0 = 340$ MeV plays the role of the constituent-quark mass, whereas our parameter $\Lambda = 340$ MeV corresponds to the effective cut-off parameter $\Lambda^{\mathrm{eff}} \approx 800$ MeV. As a result, we obtain $\langle\bar{q}q\rangle_0 = -(188\,\mathrm{MeV})^3$ and $m_{\mathrm{c}} = 13$ MeV. Remind that these values correspond to the physical pion mass.

Let us consider the π-a_1 mixing in these models. In the local NJL model the amplitude describing the π-a_1 mixing equals $A^{\mu\,(\mathrm{NJL})}_{(\pi\to a_1)} = i\sqrt{6}mp^\mu$. Therefore, the coefficient $C^{(\mathrm{NJL})}_{(\pi\to a_1)}$ equals 680 MeV. This value is 3.5 times larger than in the present model. As a result, it leads to the noticeable additional renormalization of the pion field in the local NJL model $\tilde{g}_\pi^{(\mathrm{NJL})} = g_\pi^{(\mathrm{NJL})} \cdot Z^{1/2} = m/f_\pi$, where $Z = \left(1 - 6m^2/M_{a_1}^2\right)^{-1} \approx 1.4$ and $g_\pi^{(\mathrm{NJL})} = g_\sigma^{(\mathrm{NJL})}$, in our model $Z = 1.04$. Therefore, in the local NJL model the π-a_1 mixing plays a more important role.

Let us compare also the amplitude of the decay width $\sigma \to \pi\pi$ in these models. In the local NJL model without taking into account the π-a_1 mixing in the external pion legs this amplitude equals $A^{(\mathrm{NJL})}_{\sigma\to\pi^+\pi^-} = 4m\tilde{g}_\pi^{(\mathrm{NJL})} Z^{1/2} = 4$ GeV. This amplitude is 2.4 times larger than in the present model. However, after taking into account the π-a_1 mixing this amplitude takes the form $A^{(\mathrm{NJL})}_{\sigma\to\pi^+\pi^-} = 4m\tilde{g}_\pi^{(\mathrm{NJL})} Z^{-3/2} = 2$ GeV. This is close to the amplitude obtained in our work. This leads to a noticeable decrease in the decay width $\Gamma_{\sigma\to\pi\pi} = 190$ MeV which also becomes smaller than the experimental data and is in qualitative agreement with the result of our model. The mass of the σ-meson in NJL is 570 MeV and approximately 30% larger than the result obtained here. Both the values do not contradict the experimental data.

The decay $\rho \to \pi\pi$ in [2] is used for fitting model parameters, while in our model we predict it. The mass of the a_1-meson obtained in our model practically coincides with the mass predicted in the local NJL model, $M^{(\mathrm{NJL})}_{a_1} \approx 1$ GeV.

In what follows we would like to compare our results with the nonlocal model [10]. In this model a similar separable instanton-motivated form of the interaction is also used. The main difference of our model with that of [10] is connected with an additional requirement on a quark propagator providing quark confinement. The quark mass function in our model contains only one arbitrary parameter instead of the two parameters in [10]. In spite of the minor freedom in choosing model parameters, our results are close to the results obtained in [10] (see table 1).

It is interesting also to compare our results with those obtained in [15], where the quark propagator is expressed through the entire functions which are similar to the function used in our work (see eq. (9)). The quark interaction in this work is of a separable type which is obtained from the quark-gluon interaction with the help of a modified gluon propagator. In this work, the decays $\rho \to \pi\pi$ and $a_1 \to \rho\pi$ have also been calculated. The decay ratio D/S is close to ours, while the decay widths strongly differ (see table 1).

The failure of the local NJL model and its nonlocal extensions to describe the σ-meson is expectable. Similar problems appeared in the QCD sum rule method. In the scalar channel with vacuum quantum numbers the corrections from different sources may be valuable. Indeed, it has recently been shown that the $1/N_c$ corrections in this channel are rather big [18], and the Hartree-Fock approximation may be inadequate in this case. Moreover, for a correct description of the scalar meson it is necessary to take into account the mixing with the four-quark state [19] and the scalar glueball [20].

In future, we plan to describe electromagnetic interactions in the framework of this model, calculate the e.m. pion radius, polarizability of the pion and consider the processes $\pi^0 \to \gamma\gamma$, $\gamma^* \to \gamma\pi$ in a wide domain of photon virtuality. We also plan to generalize this model to the $U(3) \times U(3)$ chiral group by introducing new parameters: the mass of the strange quark m_s and the cut-off Λ_s which allows us to describe intrinsic properties and interactions of strange mesons.

The authors thank A.E. Dorokhov for the fruitful collaboration, and D. Blaschke, C.D. Roberts and V.L. Yudichev for useful discussions. The work is supported by RFBR, grant No. 02-02-16194 and the Heisenberg-Landau program.

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